$a^2bmv^2=rr_1(br+a)f_1+rr_1(br_1+a)f$ .... (4).

From (3) and (4) since $x=(r_1-r)/2e$,

$$\frac{f_1r}{a^2r-b^2r_1}=\frac{fr_1}{a^2r_1-b^2r}=\frac{abmv^2}{2[(a^2+b^2+a^2be^2)rr_1-2a^2b^2]}.$$

II. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

Let ρ=the radius of curvature of the curve at any position, P, of the particle, ϕ=the angle included by either SP or HP, S and H being the foci, and $2a$ =the major axis. Let CD be the semi-diameter conjugate to CP, C being the center of ellipse.

The normal components of the central attractions must together equal the centrifugal force. We may assume the forces in S and H as proportional to some power of CD ; and if the absolute intensities of the two forces are equal, say μ,

$$[\mu(CD)^n+\mu(CD)^n]\cos\phi=v^2/\rho \quad\text{..................(1).}$$

But $\rho\cos\phi=\dfrac{SP.HP}{a}=\dfrac{CD^2}{a}$ ....(2),

and (1) is $2\mu(CD)^n.\dfrac{CD^2}{a}=v^2$ ....(3).

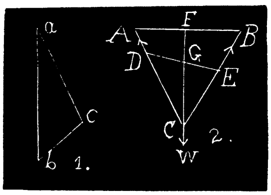
For v=a constant, requires that $n=-2$, showing plainly that the forces vary inversely as the product of the focal distances of the particle.

63. Proposed by A. H. BELL, HILLSBORO, Ill.

From a horizontal support at a distance of ten feet apart, a beam 5 feet long and 10 pounds weight is suspended by ropes attached to each end. The ropes are 3 and 5 feet respectively, in length. Required the angles made by the ropes and horizontal support. Also the stress upon each rope.

Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Regarding the bar DE as uniform so that the middle point is the center of gravity, we must have, for equilibrium, the three forces AD, BE, GC passing through the same point C, with GC perpendicular to AB.



The $\angle ABC=\angle CDE$, and the $\angle BAC=\angle CED$.

Let $\angle ABC=\theta$, $\angle BAC=\varphi$, stress on $BE=R$, on $AD=R_1$, weight of $DE=W=10$ pounds. Also $AB=10$, $BE=ED=5$, $AD=3$, $AE^2=100+25-100\cos\theta=25+9+30\cos CDE$.

$\therefore$ $130\cos\theta=91$. $\therefore$ $\cos\theta=\frac{91}{130}=.70000$. $\therefore$ $\theta=45°\ 34'\ 23''$.

$DB^2=100+9-60\cos\varphi=25+25+50\cos CED$.

$\therefore$ $110\cos\varphi=59$. $\therefore$ $\cos\varphi=\frac{59}{110}=.53636$. $\therefore$ $\varphi=57°\ 33'\ 50''$.

Let figure 2 represent the force diagram, $ab=W$, $bc=R$, $ac=R_1$, $\angle abc=90^\circ-\theta$, $\angle bac=90^\circ-\varphi$, $\angle acb=\theta+\varphi$.

$\therefore$ $R=W\cos\varphi/\sin(\theta+\varphi)$, $R_1=W\cos\theta/\sin(\theta+\varphi)$.

$\therefore$ $R=5.5078$ pounds, $R_1=7.1881$ pounds.

DIOPHANTINE ANALYSIS.

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61. Proposed by SYLVESTER ROBBINS, North Branch Depot, New Jersey.

Investigate that infinite series of prime, integral, rational scalene triangles where the sides of every term are consecutive numbers; then take the necessary factors from the proper KEY, and by an expeditious method, find in their order the areas of ten initial terms.

Solution by the PROPOSER.

I. The KEY to this series of rational triangles is $\sqrt{3}=\frac{1}{1}$, $\frac{2}{1}$, $\frac{5}{3}$, $\frac{7}{4}$, $\frac{19}{11}$, $\frac{26}{15}$, $\frac{71}{41}$, $\frac{97}{56}$, $\frac{265}{153}$, $\frac{362}{209}$, $\frac{989}{571}$, $\frac{1351}{780}$, $\frac{3691}{2131}$, $\frac{5042}{2911}$, $\frac{13775}{7953}$, $\frac{18817}{10864}$, $\frac{51409}{29681}$, $\frac{70226}{40545}$, $\frac{191861}{110771}$, $\frac{262087}{151316}$, etc. Regard the mean side as the base, and drop perpendicular from the opposite angle. Let $x=\frac{1}{2}$base. Notice that $x-2$ and $x+2$ are the segments of the base, and $\sqrt{[3(x^2-1^2)]}$ is the altitude of the triangle. Find such values for x as will render $\sqrt{[3(x^2-1^2)]}$ rational.

When $x=1$, 2, 7, 26, 97, 362, 1351, 5042, 18817, 70226, 262087, $\sqrt{[3(x^2-1^2)]}=0$, 3, 12, 45, 168, 627, 2320, 8733, 32592, 121635, 453948, etc.

These values of x are the half-bases of the several triangles. They are also the numerators of the even convergents in the expansion of $\sqrt{3}$. The values of $\sqrt{[3(x^2-1^2)]}$ are the altitudes of the same triangles, respectively, and they are also three times the denominators of the even convergents in the expansion of $\sqrt{3}$. Multiply one-half the base of a triangle by its perpendicular height, or, three times the product of the terms of the nth even convergent, must give the area of the nth triangle in the series.

Thus, $3\times2\times1=6$; $3\times7\times4=84$; $3\times26\times15=1170$; $3\times97\times56=16296$; $3\times362\times209=226974$; $3\times1351\times780=3161340$; $3\times5042\times2911=44031786$; $3\times18817\times10864=613283664$; $3\times70226\times40545=8541939510$; $3\times262087\times151316=118973869476$, etc.

II. Numerators of even convergents in expansion of $\sqrt{3}$: 1, 2, 7, 26, 97, 362, 1351, 5042, 18817, 70226, etc. Then $\frac{1}{8}(7^2-1^2)=6$; $\frac{1}{8}(26^2-2^2)=84$; $\frac{1}{8}(97^2-7^2)=1170$; $\frac{1}{8}(362^2-26^2)=16296$; $\frac{1}{8}(1351^2-97^2)=226974$; etc.

III. Denominators of even convergents : 1, 4, 15, 56, 209, 780, 2911, etc. $\frac{3}{8}(4^2-0)=6$; $\frac{3}{8}(15^2-1^2)=84$; $\frac{3}{8}(56^2-4^2)=1170$; $\frac{3}{8}(209^2-15^2)=16296$; $\frac{3}{8}(780^2-56^2)=226974$; etc.

IV. Let $x=$the half-sum of the three sides of the triangle. Then $\frac{1}{3}x-1$, $\frac{1}{3}x$ and $\frac{1}{3}x+1$ are the remainders.

$(x)[(\frac{1}{3}x)-1](\frac{1}{3}x)[(\frac{1}{3}x)+1]=$square of triangle. $3(x^2-3^2)=$square.

$\sqrt{\{(\frac{1}{3}x^2)[(x^2-3^2)/9]\}}/x=\frac{1}{3}\sqrt{[(x^2-3^2)/3]}$, the radius of inscribed circle.

Put $x=y+6$; then $3[(y+6)^2-3^2]=$square$=(my+9)^2$.

$3y^2+36y+81=m^2y^2+18my+81$; $3y+36=m^2y+18m$. $y=(18m-36)/(3-m^2)$; and $x=y+6=(18m-18-6m^2)/(3-m^2)$.